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1. Introduction. Reference is made to the paper by S. J. Kahan, "*Eight Blocks to Madness*"—*A Logical Solution*, published in the issue for March 1972 of thi MAGAZINE. Eight cubical blocks, each having sides of six different colors, are to be fitted together to form an outer cube, such that for each face of the outer cube, the four square faces which form the face are of the same color, and such that the six faces also are of the six different colors. Any particular choice of eight (repetitions allowed) of the thirty possible types of blocks (see section 2) will be referred to as a *puzzle*. We offer here general results relating to the class of puzzles, including particularly a Chart of Incompatibilities, and a List of Ground Layers, by means of which short work is made of finding the solution or solutions of any given puzzle. To illustrate the general procedure, and to illuminate the large variety of kinds of puzzles, in section 5 several examples are treated, including the specific puzzle which was discussed by Kahan, and a puzzle which has six different outer cubes (o.c.'s) as possible solutions. It is shown that no puzzle can have more than six o.c. solutions.

2. Thirty possible types of blocks or dice. A type of block is described by specifiying the colors of its Ceiling (C), Ground (G), Left face (L), Right face (R), Front (F), and Back (B). By rotation, each possible type of block may be placed so that it has a given fixed one of the six colors for C. Then there are five possible colors for G. Then for each color of G, there are six possible trace squares of a horizontal plane with the block, with edges of the four remaining colors. Therefore it is clear that there are thirty different types of blocks (and of o.c.'s).

With Kahan, we choose as six definite colors for the faces of the block, white, yellow, blue, green, purple, red ($w, y, b, g, p, r,$), and to facilitate designation of the various types, we let each cube be rotated so that it has its white face in the ceiling position. In Kahan's list (*loc. cit.*, p. 59), types 1 through 6 have ground yellow, 7 through 12 purple, 13 through 18 red, 19 through 24 green, and 25 through 30 blue. If we indicate the color of ground by a corresponding capital letter, and L, R and F, B colors by respective juxtaposed small letters, then Kahan's thirty types in numerical order are $Y(pg, rb)$, (pg, br), (pb, gr), (pb, rg), (pr, bg), (pr, gb), $P(yg, br)$ (yg, rb), (yb, rg), (yb, gr), (yr, bg), (yr, gb), $R(gb, py)$, (gb, yp), (gy, bp), (gy, pb), (gp, by), (gp, yb), $G(rb, py)$, (rb, yp), (ry, bp), (ry, pb), (rp, by), (rp, yb), $B(ry, gp)$, (ry, pg), (rp, gy), (rp, yg), (rg, yp), (rg, py). Successive odd, even numbered types are *antitypes*, i.e., either is obtained from the other by interchange of the colors of one pair of parallel faces. The effect of interchange of colors in two pairs of parallel faces is to return to the original type. Interchange of colors in three pairs of course yields the antitype.

For reasons of symmetry, we designate the thirty types, consisting of fifteen antitypical pairs of blocks, as follows. If, e.g., Y_i is a type, then Y_i' is its antitype, and the antitype of Y_i' is Y_i. Our types are Y_1 = Kahan no. 1 = $Y(pg, rb)$, Y_2 = Kahan no. 4 $= Y(pb, rg)$, $Y_3 =$ no. 6 $= Y(pr, gb)$, $P_1 =$ no. 9 $= P(yb, rg)$, $P_2 =$ no. 7 $= P(yg, br)$, $P_3 =$ no. 12 $= P(yr, gb)$, $R_1 =$ no. 13 $= R(gb, py)$, $R_2 =$ no. 16 $= R(gy, pb)$, $R_3 =$ no. 17 $= R(gp, by)$, $G_1 =$ no. 22 $= G(ry, pb)$, $G_2 =$ no. 20 $= G(rb, yp)$, $G_3 =$ no. 23 $= G(rp, by)$, $B_1 =$ no. 28 $= B(rp, yg)$, $B_2 =$ no. 29 $= B(rg, yp)$, $B_3 =$ no. 26 $= B(ry, pg)$, and the antitypes of each of the fifteen foregoing, designated by the corresponding primed symbols. For quick identification of each of the thirty types, reference may be made to the outer squares in Figure 2, below. The outer squares are the trace squares for Y_1 through B_3, in the same order as that in which they have just been listed.

Since each type is a mirror-image of its antitype, it is clear that no type of block will fit, at any corner, into its antitype as an o.c. Thus the antitypes of the blocks in any puzzle all are excluded as possible o.c.'s for a solution. Say that two blocks in an assembled o.c. are in *facially adjacent* position if they have a face in common. In any solution of a puzzle which contains an antitypical pair of blocks, the pair cannot be in facially adjacent position. For if they were, the o.c. would have to have two sides of the same color, which is not allowed.

Generally if two types are such that one as a block will not fit, at any corner, into the other as o.c., then the types will be called *incompatible*. Clearly incompatibility is a mutual relationship. The presence of a type of block in a puzzle excludes all incompatible types as possible o.c.'s for a solution. A necessary condition for the existence of a solution is that there be at least one type of o.c. with which all eight blocks are compatible.

3. Puzzles without solution, and with one or two solutions. It is obvious that a puzzle which consists of eight copies of one identical type of block is trivial: there is one and only one type of outer cube for a solution, namely the common type of the eight blocks.

THEOREM 1. *Any puzzle which contains three or more copies of one type uniquely must have that type of o.c. for any possible solution.*

Proof. Refer to the two possible diagonal locations of two blocks as *facially diagonal* (e.g., if both are in diagonal position within the ceiling layer of four blocks), and *cubically* (or *spatially*) *diagonal*. Obviously if two blocks are in either facially adjacent or cubically diagonal position, the placement of a third block in the o.c. will cause two of the three to be in facially diagonal position. The requirement that the faces of the o.c. be of the six *different* colors forces the type of the o.c. for any possible solution to be the common type.

COROLLARY 2. *Any puzzle which contains three copies of each of two different types of block has no possible solution.*

For of course the o.c. for a solution cannot simultaneously be of two different types. Corollary 2 implies that any puzzle which contains only three different types of blocks has no solution unless at least four of the blocks are of one of the types. Distributions 1, 3, 4 and 2, 3, 3 have no solution; distribution 2,2,4 can have at most one o.c. for a solution, the one of the type of the four like blocks.

COROLLARY 3. *If a puzzle contains three like blocks, and any block of type incompatible with the common type of the three (e.g., the antitype), then the puzzle has no solution.*

As shown by Kahan, the specific puzzle $(G_2, R_3, B'_3, P'_2, R'_2, P'_3, G_3, B_2)$ has only one solution for o.c., (with two different possible placements of the blocks in the o.c.). By consideration of trace squares, it may be seen that any puzzle which consists of four pairs of different Y_i's and Y_i''s (e.g., Y_1, Y_1, Y_2, Y_2, Y_3, Y_3, Y_1', Y_1') has at least two different solutions for o.c., and it may be shown that such a puzzle has exactly two solutions. (The author also has found puzzles which have exactly three, four, five, and six different o.c. solutions. Some of these will be exhibited in section 5.)

4. The one-parallel and no-parallel relations. For the thirty types of blocks, we define an irreflexive, symmetric (partial) relation as follows. Two different, non-antitypical blocks are *one-parallel* if they have one pair of parallel faces of the same colors, e.g., if they can be placed so that both have white C, yellow G; or so that both have red L, blue R. Otherwise the pair of blocks is in the *no-parallel* relation.

THEOREM 4. *If a block is in the one-parallel relation to an o.c., then it fits that o.c., but only in either of two facially adjacent positions.*

Proof. Suppose, e.g., that the block and the o.c. have the same colors for C and G. Two different nonantitypical trace squares fit each other at one and only one corner. Therefore the block fits the o.c. only along that corner, in either the C or G layer.

THEOREM 5. *If a block is in the no-parallel relation to an o.c., then either it is incompatible with that o.c. (i.e., fits at no corner of the o.c.), or it fits into the o.c. only in either of two cubically diagonal positions. In the former case, the antitypical block will fit the o.c. in either of two cubically diagonal positions. One and only one of an antitypical pair of blocks is compatible with a no-parallel related outer cube.*

Proof. Comparison of Y_1 with any no-parallel related block, e.g., P_3, shows either that at a corner where the three colors are common, in one they are clockwise (c.) and in the other counterclockwise (c.c.) (e.g., the respective c. and c.c. *wgr* corners of Y_1 and P_3), or that they agree at a corner (e.g., at c. *wgr* for Y_1 and P_3'). At the diagonally opposite corners where the colors also agree, the situation is the same (for Y_1 and P_3, the *ybp* corner is respectively c. and c.c., while for Y_1 and P_3', both are c.). Change from c. to c.c. at only one of the two diagonally opposite corners forces the two blocks to be in the one-parallel relation.

Table 1 below presents the one-parallel relation, and also the relation of incompatibility, for the thirty types of blocks. The one-parallel relation is indicated by $\cdot$, and incompatibility by $\times$. We ask the reader to bear in mind that, e.g., $Y_1 \cdot G_2$ implies $Y_1' \cdot G_2$, $Y_1 \cdot G_2'$, $Y_1' \cdot G_2'$, and also that, e.g., $Y_1 \times P_3$ is equivalent to $Y_1' \times P_3'$, $Y_1' \times B_2$ to $Y_1 \times B_2'$. Use of these facts, and of symmetry of the relations, reduces the size of Table 1 by a factor of one-fourth.

TABLE 1

The one-parallel relation, and incompatibility

	Y_1	B_1	G_1	P_1	R_1	R_2	Y_2	B_2	G_2	P_2	P_3	R_3	Y_3	B_3	G_3	
Y_1		×	×	×	×	×	·		·	·	×	·	·	·		Y_1
B_1			×	×	×	·	×	·		·		×	·	·	·	B_1
G_1				×	×	·	·	×	·		·		×	·	·	G_1
P_1					×		·	·	×	·	·	·		×	·	P_1
R_1						·		·	·	×	·	·	·		×	R_1
R_2		·	·	×	·		·			·	×	·	×			R_2
Y_2	·		·	·	×	·		·				×	·	×		Y_2
B_2	×	·		·	·	×	·		·				×	·	×	B_2
G_2	·	×	·		·	×	×	·		·	×			×	·	G_2
P_2	·	·	×	·		·	×	×	·		·	×			×	P_2
P_3		×	·	·	·		×	×		·			·	·		P_3
R_3	·		×	·	·	·		×	×		×			·	·	R_3
Y_3	·	·		×	·		·		×	×	·	×			·	Y_3
B_3	·	·	·		×	×		·		×	·	·	×			B_3
G_3	×	·	·	·		×	×		·		×	·	·	×		G_3
	Y_1'	B_1'	G_1'	P_1'	R_1'	R_2'	Y_2'	B_2'	G_2'	P_2'	P_3'	R_3'	Y_3'	B_3'	G_3'	

As a result of (hard-won) symmetrical design by the author, the five blocks Y_1, B_1, G_1, P_1, R_1 are *mutually* no-parallel and *mutually* incompatible. Interchange of colors b, g yields the same situation for the five blocks $Y_2, G_3', B_3, P_2', R_1'$; of r, g, the same for $Y_3, B_3', R_2, P_1', G_2'$; of g, p, the same for $Y_1', B_2, P_3', G_3, R_2'$; of g, y the same for $G_2, B_1', Y_2', P_3, R_3'$; and of r, y, the same for $R_3, B_2', G_1', P_2, Y_3'$. This partitions the set of thirty (types of) blocks into six pairwise disjoint subsets, each containing five blocks. Passage to the antitypes of the blocks of each set of five yields another partitioning of the thirty blocks into six sets, each consisting of five mutually incompatible blocks. These incompatibility relations include all of the (nonantitypical) incompatibilities between the thirty blocks. All of this information is presented in the simple chart on page 120 (Figure 1).

The presence of a given block in a puzzle excludes the four other blocks in the column containing the given block, and excludes the four other blocks in the polygonal line containing the given block, as possible o.c.'s for a solution of the puzzle. As already remarked, the given block also excludes its antitype. Thus the Chart (Figure 1) is a simplified representation of the graph of the relation of incompatibility, and in graph-theoretical language, clearly that graph is a regular graph on thirty vertices, of valency nine (i.e., each type is incompatible with its antitype, and with eight other types).

5. Method for solution of any puzzle. The following list of fifteen squares, with marked subsquares, presents information gained by use of Table 1, in conjunction with Theorems 4 and 5, and with the description of the thirty types in section 2. With the types in the standard positions as in section 2, the outer squares in the five lines in left to right; top to bottom order represent the *Ground* layers of, respectively, Y_1, Y_2, Y_3; P_1, P_2, P_3; R_1, R_2, R_3; G_1, G_2, G_3; B_1. B_2, B_3. For each o.c., the blocks which will fit into the ground layer and where, are indicated by the marked types on the subsquares. The squares represent the *Ceiling* layers for o.c.'s which are of the antitypes Y_1' Y_2', etc,, provided it is understood (i) that the antitypical o.c.'s are in inverted position so that ceiling and ground colors are interchanged (thus preserving the indicated colors of the vertical sides), and (ii) that each type marked on a subsquare is replaced by its antitype. The lower two blocks in a corner represented by a subsquare also fit in the *cubically diagonally located* corner of the other layer (ground or ceiling). The antitypes of the upper three blocks in the subsquare are those which fit in the cubically diagonally located corner. The upper three blocks are one-parallel with the type which is represented by the outer square; the other two blocks are no-parallel related with that type.

Now we are ready to present our method or procedure for dealing with any puzzle. As a first example, we solve the specific puzzle which was discussed by Kahan.

STEP 0. *By reference to the outer squares of Figure* 2, *determine which types of blocks are included in the given puzzle, and how many of each.*

By examination and comparison of the blocks of the specific puzzle with Figure 2, it is determined that they are P_2', P_3', R_2', R_3, G_2, G_3, B_2, B_3'.

STEP 1. *In case the puzzle contains three or more copies of one type X, then by Theorem* 1, *X is the only possible o.c. for a solution. If the puzzle also contains one X', or three copies of another type Y (as it would, e.g., if only two types were represented in the puzzle), then by Corollaries* 2–3, *there is no solution. If there is neither an X' nor three Y's, then refer to the Chart (Step* 2) *to see if X is excluded as o.c. by another block in the puzzle. If not, proceed to Step* 3. (As a consequence of the remark about distributions following Corollary 2, in order that a puzzle possibly may have more than one o.c. solution, it is necessary that it contain four or more different types.)

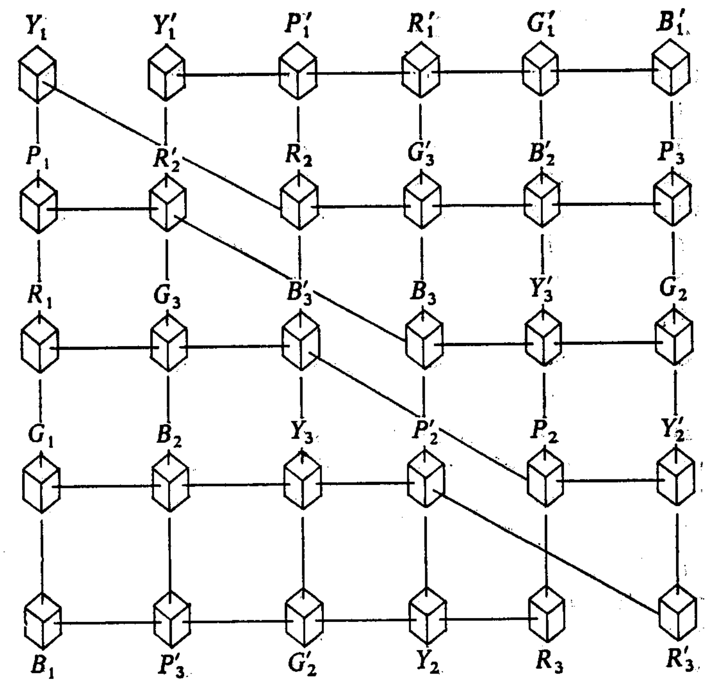

FIG. 1.

Chart of Incompatibilities

The blocks in each column of five blocks are *mutually* incompatible. The set of five joined blocks in the top row, the set of five in the bottom row, and the sets of five in the four bent rows, likewise each are mutually incompatible sets. All nonantitypical incompatibilities, one-hundred twenty in number, are included among those of the mutual incompatibilities (ten per each of the twelve sets of five). Thus the Chart is a simple representation of a regular graph of valency eight (nine if the antitypicalities are included) on thirty vertices (one-hundred thirty-five edges).

Notes for FIG. 2, **List of Ground Layers** (page 121)

The possible ground or ceiling layers, for any typical or antitypical o.c., may be derived from the squares in Figure 4. As marked, the squares represent the ground layers for Y_1, $Y_2\ Y_3$; P_1, P_2, P_3; etc. With the marked types replaced by their antitypes (e.g., Y_3 by Y_3', Y_2' by Y_2, etc.), the same squares represent the ceiling layers for o.c.'s Y_1', Y_2', Y_3', etc., in inverted position. The square for the ceiling layer of e.g.o.c. Y_1 is obtained by marking $Y_3'\ P_2'\ B_3'$ in, e.g., the subsquare diagonally opposite the subsquare which is marked $Y_3 P_2 B_3$, and $P_3'\ B_1'$ in the same diagonally opposite subsquare. The same new square, except with all marked types replaced by antitypes in the same positions, represents the ground layer of inverted o.c. Y_1'. Each type of block fits at every corner of the same type of o.c.; the types identical with the o.c. types are not marked in the subsquares.

	b		
p	$Y_3P_2B_3$ $P'_3B'_1$	$Y'_2G'_2B_3$ G'_1B_2	g
	$Y_2P_2R'_3$ $P'_1R'_2$	$Y'_3R'_3G'_2$ R'_1G_3	
	r		

	g		
p	$Y'_3P'_1G_1$ P_3G_3	$Y'_1G_1B'_2$ $G_2B'_3$	b
	$Y_1P'_1R'_2$ $P_2R'_3$	$Y_3R'_2B'_2$ $R_1B'_1$	
	r		

	b		
p	$Y_1P'_3B'_1$ P_2B_3	$Y_2R_1B'_1$ $R'_2B'_2$	r
	$Y'_2P'_3G'_3$ $P_1G'_1$	$Y'_1R_1G'_3$ R_3G_2	
	g		

	g		
y	$Y'_2P'_3G'_3$ $Y_3G'_1$	$P_2G'_3B_2$ $G'_2B'_1$	b
	$Y'_2P'_2R_3$ Y'_1R_2	$P_3R_3B_2$ $R'_1B'_3$	
	r		

	r		
y	$Y_1P'_1R'_2$ $Y_2R'_3$	$P_3R'_2G'_2$ R'_1G_1	g
	$Y_1P'_3B'_1$ Y_3B_3	$P_1G'_2B'_1$ G'_3B_2	
	b		

	b		
y	$Y'_3P'_2B'_3$ Y'_1B_1	$P_1R'_1B'_3$ R_3B_2	r
	$Y'_3P'_1G_1$ Y_2G_3	$P_2R'_1G_1$ $R'_2G'_2$	
	g		

	y		
g	$Y_3R_3G_2$ $Y'_1G'_3$	$Y_3R'_2B'_2$ $Y_2B'_1$	b
	$P'_3R_2G_2$ $P'_2G'_1$	$P'_3R'_3B'_2$ P'_1B_3	
	p		

	b		
g	$R'_3G'_1B_1$ G_3B_3	$Y'_2R'_1B_1$ Y'_3B_2	y
	$P'_2R_1G'_1$ P'_3G_2	$Y'_2P'_2R_3$ Y'_1P_1	
	p		

	y		
g	$Y'_1R_1G'_3$ Y_3G_2	$Y'_1P_1R_2$ $Y'_2P'_2$	p
	$R'_2G'_3B'_3$ $G_1B'_1$	$P_1R'_1B'_3$ P_3B_2	
	b		

	b		
r	$R'_2G'_3B'_3$ $R_3B'_1$	$Y_2G_2B'_3$ $Y'_1B'_2$	y
	$P_3R'_2G'_2$ $P_2R'_1$	$Y_2P_3G_3$ $Y'_3P'_1$	
	p		

	p		
r	$P'_2R_1G'_1$ P'_3R_2	$P'_2G_3B'_2$ P'_1B_1	b
	$Y'_1R_1G'_2$ Y_3R_3	$Y'_1G_1B'_2$ $Y_2B'_3$	
	y		

	y		
r	$Y'_3R'_3G'_2$ $Y_1R'_1$	$Y'_3P'_1G_1$ Y_2P_3	p
	$R'_3G'_1B_1$ R_2B_3	$P'_1G_2B_1$ $P'_2B'_2$	
	b		

	g		
r	$R_2G_3B_3$ $R'_3G'_1$	$P'_2G_3B'_2$ P'_1G_2	p
	$Y'_3R_2B_2$ $Y'_2R'_1$	$Y'_3P'_2B'_3$ Y'_1P_3	
	y		

	p		
r	$P_1R'_1B'_3$ P_3R_3	$P_1G'_2B'_1$ $P_2G'_3$	g
	$Y'_2R'_1B_1$ $Y'_3R'_2$	$Y'_2G'_2B_3$ $Y_1G'_1$	
	y		

	g		
r	$R'_3G'_1B_1$ R_2G_3	$Y_1G'_1B_2$ $Y'_2G'_2$	y
	$P'_3R'_3B'_2$ P'_1R_1	$Y_1P'_3B'_1$ Y'_3P_2	
	p		

In the specific puzzle, there is no repetition of types. Therefore in order to see which types are possible o.c. solutions for it, proceed to Step 2.

STEP 2. *Mark five rows of six dots, as in the chart (Figure 1) on a piece of paper, and circle those corresponding to the types which occur in the puzzle. Draw in the vertical and the polygonal lines of the chart which pass through the circled dots. If two or more circled dots are in a line then all types in that line are excluded. In case of only the one circled dot in both of the lines through it, the corresponding type is a possible o.c. unless it is excluded by an antitype. Mark × through any antitypes which are not already excluded by the drawn-in lines.*

Following (Figure 3) is the result of the procedure of Step 2 applied to the specific puzzle. All eight antitypes already are excluded by the other incompatibilities, so in this case of the specific puzzle, it is not necessary to mark any antitypes.

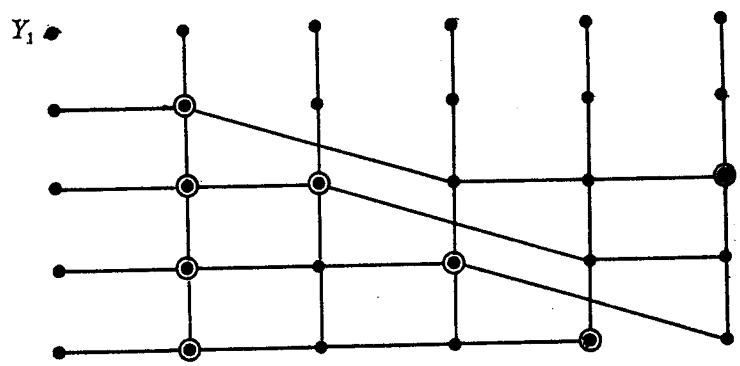

FIG. 3. **Chart for the specific puzzle.**

This shows that all o.c.'s except Y_1 are excluded so that o.c. Y_1 is the only possible solution.

It is obvious that Y_1 is the only possible o.c. solution for the specific puzzle. (It is also obvious that the specific puzzle contains four mutually incompatible types, and that any puzzle which contains five mutually incompatible types has no solution. However if, e.g., the five types are those in the second column then the exclusion of Y_1, R_2, G_3', B_2', P_3 is only by the fact that they are the antitypes of the types in the second column.)

STEP 3. *For each possible type of o.c. which is yielded by the chart, by using the List of Ground Layers, sketch the outer squares for the ground and ceiling layers, marking down in the appropriate subsquares those types which are included in the puzzle. If a simultaneous placement of the eight blocks in the eight subsquares* *is possible, then that o.c. is a solution, with the indicated placement.* Otherwise of course the o.c. is not a solution in spite of the individual compatibility of the blocks with the o.c. For a given o.c. solution there may exist several alternative placements.

Applied to the specific puzzle, Step 3 produces the three squares in Figure 4. Placement in the ground layer is unique; the other two squares represent the possible alternative placements in the ceiling layer.

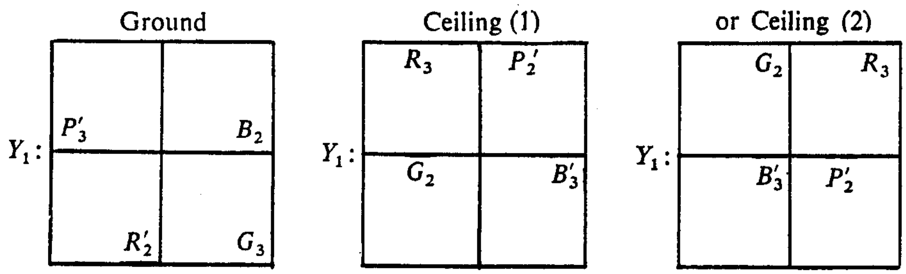

FIG. 4. **Placements of the blocks for the specific puzzle (Example 1).**

EXAMPLE 2. Consider puzzle $\mathscr{P}_2 = (Y_2, Y_3', P_1', P_2, R_2', G_2', B_1', R_3')$. As in the case of the specific puzzle $\mathscr{P}_1$, use of the chart shows that Y_1 is the only possible o.c. for a solution. See Figure 5. But reference to the list reveals that no block of $\mathscr{P}_2$ fits in subsquare (or corner) CLB of o.c. Y_1. Therefore $\mathscr{P}_2$ has no solution. There are, however, several simultaneous and compatible placements of seven of the blocks in o.c. Y_1.

EXAMPLE 3. Consider puzzle $\mathscr{P}_3 = (P_2, P_2, R_2', G_1', G_2, G_3, G_3, B_1')$. This puzzle contains six different types, but does not contain three of a common type. Therefore we proceed to Step 2. The marked chart is as follows. See Figure 6. All o.c.'s evidently are excluded, except Y_1, Y_2, Y_3. Proceeding to Step 3, we find that in fact each of Y_1, Y_2, Y_3 is a solution of $\mathscr{P}_3$. The ground and ceiling layers are as follows. See Figure 7. In the case of each of the three o.c. solutions Y_1, Y_2, Y_3, the placement of the blocks is uniquely determined.

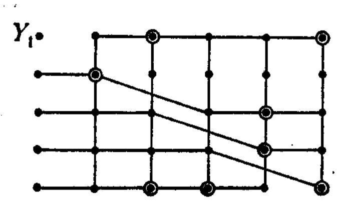

FIG. 5
Chart for Puzzle $\mathscr{P}_2$

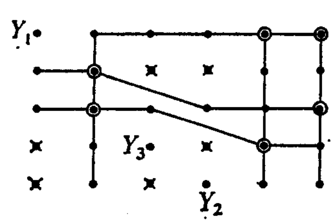

FIG. 6
Chart for Puzzle $\mathscr{P}_3$

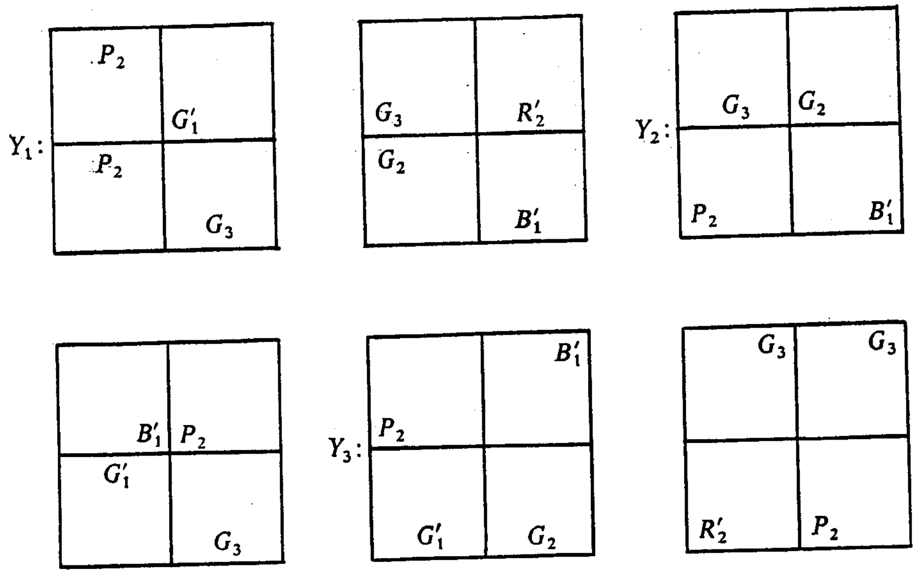

FIG. 7
Unique placements for the three o. c. solutions of $\mathscr{P}_3$.

For further examples, the reader easily may determine, using the chart and the list, that puzzles $\mathscr{P}_4 = (Y_3, P_3', R_1, G_3, B_1', B_1', B_2', B_3)$, $\mathscr{P}_5 = (Y_2, P_2', R_3, R_3, R_3', R_3', G_1', B_1')$, $\mathscr{P}_6 = (Y_1, Y_1, G_1, G_1, B_1, B_1, B_2', P_3)$, have respectively four, five, and six o.c. solutions. The chart shows that puzzle $\mathscr{P}_7 = (Y_2, Y_2, R_3, R_3, R_3', G_1', B_1', B_1')$ is compatible with seven o.c.'s as solutions, but the list shows that for all o.c.'s except Y_1, there is some corner at which no block of the puzzle fits. Therefore $\mathscr{P}_7$ has Y_1 as its only o.c. solution.

THEOREM 6. *There does not exist any puzzle which has more than six different o.c.'s as solutions.*

Editorial demand that the author cut the length of this paper forces omission of the proof, which requires exhaustion of twenty-four cases, and is seven pages in length. The author will supply a copy of the seven pages on request. Perhaps a reader will be able to construct a more elegant proof.

Reference

1. S. J. Kahan, Eight blocks to madness, a logical solution, this MAGAZINE, 45 (1972) 57–72.