RESEARCH PROBLEMS

8. Andrew Sobczyk: *Projections in Banach spaces.*

Does there exist any infinitely-dimensional, separable, closed linear subspace X of the nonseparable space (m) of all bounded sequences, such that there is a continuous projection of (m) onto X? Phillips and Sobczyk have shown that there is no continuous projection of (m) onto (c_0), the subspace of all sequences convergent to zero. Sobczyk has shown that for any separable closed linear subspace W, $W \supset (c_0)$, there is a projection of bound 2 of W onto (c_0), and therefore no continuous projection of (m) onto W. A lemma of Murray states that there is a continuous projection onto a closed linear subspace Y if and only if there is a complementary closed linear subspace Z. For any Banach space $U \supset (m)$, there is a projection of bound 1 onto (m). Does there exist a pair of complementary closed linear subspaces Y, Z for (m), such that neither Y nor Z is separable or isomorphic with (m)? Similar questions may be asked concerning the existence of closed projections. References: D. B. Goodner, Trans. Amer. Math. Soc. vol. 69 (1950) pp. 89–108. F. J. Murray, Bull. Amer. Math. Soc. vol. 48 (1942) pp. 76–93. R. S. Phillips, Trans. Amer. Math. Soc. vol. 48 (1940) pp. 516–541. A. Sobczyk, Bull. Amer. Math. Soc. vol. 47 (1941) pp. 938–947; Duke Math. J. vol. 8 (1941) pp. 78–106; Trans. Amer. Math. Soc. vol. 55 (1944) pp. 153–169. (Received February 11, 1954.)

9. Lowell J. Paige: *Elements of odd order in a finite group.*

Let G be a finite group of order $n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k} \cdot 2^s$, where $p_1, p_2, \cdots, p_k$ are distinct odd primes. Let P be a Sylow 2-subgroup of G and let S be the set of all elements of G satisfying the equation $x^r = 1$, where $r = p_1^{\alpha_1} \cdots p_k^{\alpha_k}$. For the coset expansion of G by P,

$$G = g_1 P + g_2 P + \cdots + g_r P,$$

is there an element of S in each coset $\{g_i P\}$ $(i = 1, 2, \cdots, r)$? Note that the problem is trivial if P is normal or if P is its own normalizer in G and the intersection of P with each of its conjugates is the identity. (Received February 15, 1954.)

10. Casper Goffman: *The group of similarity transformations of a simply ordered set.*

Let S be a simply ordered set. A similarity transformation of S is a one-to-one correspondence between S and itself which preserves order. The similarity transformations of S form a group $G(S)$. For a well ordered set S, $G(S)$ consists of a single element, but there are other ordered sets with this property. The problem is the following: (a) characterize the ordered sets S for which $G(S)$ consists of one element; (b) characterize the ordered sets S for which $G(S)$ is abelian; (c) characterize the groups G for which there is an ordered set S such that $G = G(S)$. (Received February 17, 1954.)

11. O. Taussky: *Multiply monotonic sequences.*

Szegö (Duke Math. J. vol. 8 (1941) pp. 559–564) investigated the following theorem of Fejér: Let the sequence $\{a_n\}$ be monotonic of order 4. Then the power series $f(z) = \sum_{n=0}^{\infty} a_n z^n$ is regular and univalent for $|z| < 1$. Szegö proved that this theorem remains true for monotonic sequences of order 3, but is not true for monotonic sequences of order 2. Investigate what happens to monotonic sequences of order α with $2<\alpha<3$. (For multiply monotonic sequences of non-integral order see K. Knopp, Math. Zeit. vol. 22 (1925) pp. 75–85). (Received March 1, 1954.)

12. O. Taussky: *The Hilbert matrix.*

Denote by H the infinite matrix

$$\left(\frac{1}{i+k}\right).$$

Let $x=(x_1, x_2, \cdots)$ be a vector with $x'x=\sum x_i^2<\infty$. Hilbert proved that $x'Hx/x'x<\pi$. From this fact it follows at once that the system of equations $Hx'=\pi x'$ cannot be solved. It seems of interest to find out whether $Hx'=\pi x'$ can be solved if x is an arbitrary vector as long as the product Hx' exists. (For relevant recent literature see, e.g., W. Magnus, Amer. J. Math. vol. 72 (1950) pp. 699–704, Archiv d. Math. vol. 2 (1951) pp. 405–412; O. Taussky, Quart. J. Math. Oxford Ser. vol. 20 (1949) pp. 80–83.) (Received March 1, 1954.)

13. L. C. Young: *Parallel polyhedral surfaces.*

The two-dimensional oriented polyhedra π_1 and π_2 situated in Euclidean n-space are termed *parallel* if they can be decomposed into finite sums $\pi_1=\sum T_{1n}$, $\pi_2=\sum T_{2n}$ where T_{1n} and T_{2n} are triangles derivable from one another by translation. More generally, π_1 and π_2 are parallel outside area ϵ if they can be expressed in the form $\pi_1=\pi_1'+\pi_1''$, $\pi_2=\pi_2'+\pi_2''$, where π_1' and π_2' are parallel and $\pi_1''+\pi_2''$ has area $\leqq\epsilon$. The following question seems to have an important bearing on surface-integral problems of the calculus of variations: If we suppose π_1 closed and $\epsilon>0$ given, does there always exist a corresponding π_2 *of the type of the sphere,* such that π_1 and π_2 are parallel outside area ϵ? (Received March 18, 1954.)

14. P. L. Butzer: *Tauberian conditions.*

In the theory of divergent series, Tauberian theorems assert that any sequence which is summable by a definite method of summation and which satisfies an appropriate additional condition (τ) is necessarily convergent. The condition (τ) is said to be the *Tauberian condition* for the method of summation in question. *Conjecture*: There exist conditions (τ) which are Tauberian conditions for the Cesàro but not the Abel method. Likewise we may ask whether there are conditions (τ) which are Tauberian for the Lambert (see G. H. Hardy, *Divergent series*, Oxford, 1949, p. 372) but not the Abel method. (Received March 24, 1954.)

15. Richard Bellman: *Stability theory.*

It is known that if all the solutions of the vector-matrix equation $dy/dt=A(t)y$ are bounded as $t\to\infty$, then all the solutions of the perturbed equation $dz/dt=(A(t)+B(t))z$ are bounded as $t\to\infty$, provided that $\int^\infty\|B(t)\|dt<\infty$, in the two cases where $A(t)$ is constant or periodic. Does the result hold if $A(t)$ is merely restricted to be almost-periodic, or, in particular, have as elements finite trigonometric sums? (Received March 24, 1954.)

16. Richard Bellman: *Number theory.*

Let the integer n be written in the dyadic scale, $n=2^{k_1}+2^{k_2}+\cdots+2^{k_r}$, with $k_1>k_2>\cdots>k_r\geqq 0$, and define the number-theoretic function $\alpha(n)=r$. Is it true that there exist infinitely many primes for which $\alpha(n)$ is less than some fixed integer? (Received March 24, 1954.)

17. Richard Bellman: *Matrix theory.*

Let $\{A_i\}$, $i=1, 2, \cdots, k$, be a finite set of positive square matrices, and let $S_N=\{\prod_i B_i\}$ be the set of k^N matrices obtained by taking all possible products $B_1B_2 \cdots B_N$ where each B_i is an A_j. For any positive matrix X, let $\phi(X)$ denote the characteristic root of X of largest absolute value. A classical result of Perron asserts that $\phi(X)$ is positive. Let C_N be a matrix in the set S_N for which this root is a maximum. It is easy to show that $\lambda=\lim_{N\to\infty}\phi(C_N)^{1/N}$ exists. Let M_N denote the smallest majorant of the set S_N, that is, the ijth element of M_N is the maximum of the k^N ijth elements of matrices in S_N. Then again it is easy to show that $\mu=\lim_{N\to\infty} \phi(M_N)^{1/N}$ exists. Is it true that $\lambda=\mu$? (Received March 29, 1954.)

18. Richard Bellman: *Systems of renewal equations.*

Let $\{\phi_{ij}(t)\}$, $i, j=1, 2, \cdots, N$, be a matrix of non-negative functions, all of whose Laplace transforms possess a finite abscissa of convergence, with the additional condition that $\int_0^\infty \phi_{ii}dt>1$ for some i. It has been shown by Bohnenblust that the root of $\left|\int_0^\infty e^{-st}\phi_{ij}dt-I\right|=0$ with largest real part is positive. That it is also simple is trivial for $N=1$, and readily demonstrated for $N=2$. Is this root simple for $N\geqq 3$? (Received March 29, 1954.)