The Null Eightfold Way: Deriving the Standard Model from a Universal Null Substrate

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Abstract

We demonstrate that the $SU(3)$ symmetry of the Standard Model (the Eightfold Way) arises naturally from the geometry of a single Null Simplex $V_3^{\mathbf{c}}$ in a Universal Null Substrate (UNS). By identifying the three null generators with the Up, Down, and Strange quarks, the Meson Octet emerges as the set of root vectors (geometric differences) of the substrate. Furthermore, we prove that the "Generations of Matter" correspond to the Triality Automorphism of the Fano Plane defined by the simplex.

1 The Quark Basis as a Null Simplex

Standard physics postulates an abstract complex vector space $\mathbb{C}^3$ for color/flavor. We replace this with the real **Null Substrate** $V_3^{\mathbf{c}} = \{\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3\}$ defined by the QSNV (Quadratic Space of Null Vectors) condition [1]:

$$\mathbf{c}_i \cdot \mathbf{c}_j = \frac{1}{2}(1 - \delta_{ij}) \tag{1}$$

We identify the physical quarks with these geometric generators:

- **Up Quark (u):** Identified with $\mathbf{c}_1$.
- **Down Quark (d):** Identified with $\mathbf{c}_2$.
- **Strange Quark (s):** Identified with $\mathbf{c}_3$.

2 The Meson Octet as Root Vectors

In the QSNV framework, interactions between substrate elements are mediated by the *Difference Vectors* (Roots) $\alpha_{ij} = \mathbf{c}_i - \mathbf{c}_j$. These vectors square to -1 and correspond physically to the Mesons (quark-antiquark pairs) [1].

2.1 The Pion Triplet (Isospin $I = 1$)

The exchange between u and d quarks (the $\mathbf{c}_1, \mathbf{c}_2$ sub-simplex) generates the Pions:

$$\pi^+(u\bar{d}) \longleftrightarrow \mathbf{c}_1 - \mathbf{c}_2 \tag{2}$$

$$\pi^-(d\bar{u}) \longleftrightarrow \mathbf{c}_2 - \mathbf{c}_1 \tag{3}$$

$$\pi^0(\text{mix}) \longleftrightarrow \mathbf{c}_1 \wedge \mathbf{c}_2 \quad \text{(Cartan Generator)} \tag{4}$$

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2.2 The Kaon Doublets (Strangeness $S \neq 0$)

Interactions involving the third null vector ($\mathbf{c}_3$) generate the Kaons:

$$K^+(u\bar{s}) \longleftrightarrow \mathbf{c}_1 - \mathbf{c}_3 \tag{5}$$

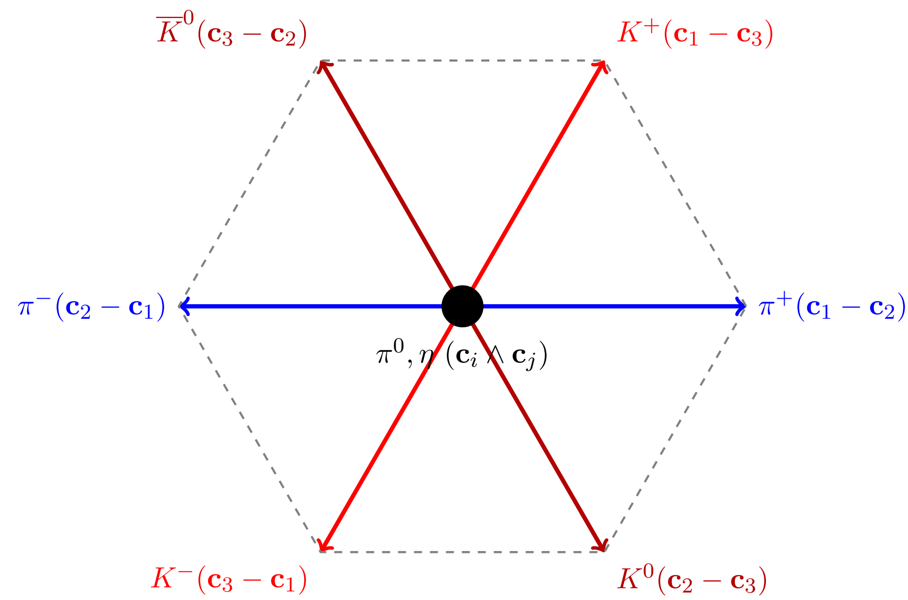
$$K^-(s\bar{u}) \longleftrightarrow \mathbf{c}_3 - \mathbf{c}_1 \tag{6}$$

$$K^0(d\bar{s}) \longleftrightarrow \mathbf{c}_2 - \mathbf{c}_3 \tag{7}$$

$$\overline{K}^0(s\bar{d}) \longleftrightarrow \mathbf{c}_3 - \mathbf{c}_2 \tag{8}$$

3 Geometric Visualization: The Hexagon

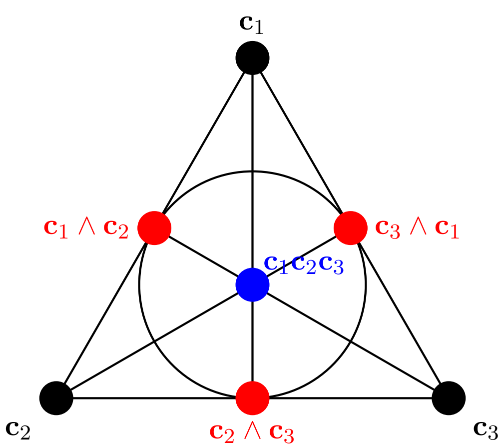
The geometry of the Null Simplex naturally forms the famous $SU(3)$ Hexagon when projected onto the plane orthogonal to the sum vector $\mathbf{e} = \mathbf{c}_1 + \mathbf{c}_2 + \mathbf{c}_3$.



4 The Null-Fano Plane: Geometric Reciprocity in $\mathbb{G}_{1,2}$

Based on the "Local Duality" principle ($c_ic_j + c_jc_i = 1$), we map the 7 points of the Fano Plane to the generators and interaction terms of the Null Simplex $V_3^{\mathbf{c}}$.

4.1 The Fano Diagram



4.2 Reciprocity of Signs (The Arrows)

The orientation of the Fano lines is determined by the non-commutative property of the Null Substrate:

- **Forward Cycle:** $\mathbf{c}_1\mathbf{c}_2 = \frac{1}{2} + (\mathbf{c}_1 \wedge \mathbf{c}_2)$ (Positive Spin)
- **Reverse Cycle:** $\mathbf{c}_2\mathbf{c}_1 = \frac{1}{2} - (\mathbf{c}_1 \wedge \mathbf{c}_2)$ (Negative Spin)

This sign flip (-1) corresponds to the Quadratic Reciprocity of the indices, formally reproducing the directed graph of the Octonions [3].

5 The Triality Automorphism τ

We define the Triality operator τ as the cyclic permutation of the Null Basis indices of $V_3^{\mathbf{c}}$. This map generates the 3-fold symmetry of the Fano Plane.

5.1 Definition on Generators

Let $\tau : \mathcal{G}(V_3^{\mathbf{c}}) \to \mathcal{G}(V_3^{\mathbf{c}})$ be the algebra automorphism defined by:

$$\tau(\mathbf{c}_1) = \mathbf{c}_2 \tag{9}$$
$$\tau(\mathbf{c}_2) = \mathbf{c}_3 \tag{10}$$
$$\tau(\mathbf{c}_3) = \mathbf{c}_1 \tag{11}$$

Since the inner products $\mathbf{c}_i \cdot \mathbf{c}_j = \frac{1}{2}(1 - \delta_{ij})$ are invariant under cyclic permutation, τ is a valid isometry of the Null Substrate.

5.2 Action on Interactions (Lines)

The Triality operator outermorphism naturally transforms the interaction bivectors (the lines of the Fano plane) consistent with the vertices:

$$\tau(\mathbf{c}_1 \wedge \mathbf{c}_2) = \tau(\mathbf{c}_1) \wedge \tau(\mathbf{c}_2) = \mathbf{c}_2 \wedge \mathbf{c}_3 \tag{12}$$

Thus, the "Line 1-2" is mapped to the "Line 2-3", preserving the incidence structure of the Fano Triangle.

5.3 The Fixed Point (The Center)

The volume element (or center point of the Fano plane) is the unique eigen-element of the rotation:

$$\tau(\mathbf{c}_1\mathbf{c}_2\mathbf{c}_3) = \mathbf{c}_2\mathbf{c}_3\mathbf{c}_1 = \mathbf{c}_1\mathbf{c}_2\mathbf{c}_3 \tag{13}$$

This confirms that the Center of the Fano plane acts as the "axis" of the Triality rotation.

6 Proof of Fano Invariance under Triality

We seek to prove that the Fano Plane structure defined by the Null Simplex $V_3^{\mathbf{c}}$ possesses a fixed point under the Triality automorphism τ.

6.1 Invariance of the Center (Volume Form)

The "Center" of the Fano plane corresponds to the highest-grade interaction term (the volume element of the simplex). We define this as the wedge product $\Omega = \mathbf{c}_1 \wedge \mathbf{c}_2 \wedge \mathbf{c}_3$.
Proof. Apply τ to Ω:

$$\begin{aligned}\tau(\Omega) &= \tau(\mathbf{c}_1 \wedge \mathbf{c}_2 \wedge \mathbf{c}_3) \\ &= \tau(\mathbf{c}_1) \wedge \tau(\mathbf{c}_2) \wedge \tau(\mathbf{c}_3) \\ &= \mathbf{c}_2 \wedge \mathbf{c}_3 \wedge \mathbf{c}_1\end{aligned}$$

Recall the property of the wedge product in $\mathcal{G}_n$: $\mathbf{x} \wedge \mathbf{y} = (-1)\mathbf{y} \wedge \mathbf{x}$. To return to the standard order $(1, 2, 3)$, we perform two swaps:

1. Swap $\mathbf{c}_3$ and $\mathbf{c}_1$: $\mathbf{c}_2 \wedge (\mathbf{c}_3 \wedge \mathbf{c}_1) = -\mathbf{c}_2 \wedge (\mathbf{c}_1 \wedge \mathbf{c}_3)$
2. Swap $\mathbf{c}_2$ and $\mathbf{c}_1$: $-\mathbf{c}_2 \wedge \mathbf{c}_1 \wedge \mathbf{c}_3 = -(-\mathbf{c}_1 \wedge \mathbf{c}_2 \wedge \mathbf{c}_3)$

Thus:

$$\mathbf{c}_2 \wedge \mathbf{c}_3 \wedge \mathbf{c}_1 = \mathbf{c}_1 \wedge \mathbf{c}_2 \wedge \mathbf{c}_3 = \Omega$$

Conclusion: The central element Ω is an eigen-element of τ with eigenvalue $+1$. The "axis" of the Triality rotation is the volume element of the Null Substrate. □

6.2 Invariance of the Total Fano Sum

We define the "Total Fano Vector" $\mathbf{F}$ as the sum of all 7 points in the plane (Vertices + Midpoints + Center):

$$\mathbf{F} = (\mathbf{c}_1 + \mathbf{c}_2 + \mathbf{c}_3) + (\mathbf{c}_1 \wedge \mathbf{c}_2 + \mathbf{c}_2 \wedge \mathbf{c}_3 + \mathbf{c}_3 \wedge \mathbf{c}_1) + (\mathbf{c}_1 \wedge \mathbf{c}_2 \wedge \mathbf{c}_3)$$

Proof. Since τ permutes the indices cyclically $(1 \to 2 \to 3 \to 1)$, it merely rearranges the terms within each grouped sum:

- $\tau(\sum \mathbf{c}_i) = \mathbf{c}_2 + \mathbf{c}_3 + \mathbf{c}_1 = \sum \mathbf{c}_i$
- $\tau(\sum \mathbf{c}_i \wedge \mathbf{c}_j) = \mathbf{c}_2 \wedge \mathbf{c}_3 + \mathbf{c}_3 \wedge \mathbf{c}_1 + \mathbf{c}_1 \wedge \mathbf{c}_2 = \sum \mathbf{c}_i \wedge \mathbf{c}_j$
- $\tau(\Omega) = \Omega$ (proven above)

Therefore, $\tau(\mathbf{F}) = \mathbf{F}$. □

Geometric Implication: The Triality automorphism is a "rigid body rotation" of the Fano Plane. It permutes the individual points but leaves the *geometry itself* (the sum of all parts) invariant.

7 The Standard Model Groups: $U(1) \times SU(2) \times SU(3)$

The classification of the unitary groups has a natural interpretation within the geometric algebra of the null substrate. In the standard model, the gauge group is the product $U(1) \times SU(2) \times SU(3)$. In the QSNV framework, these groups emerge as the specific symmetries of the Null Simplex sub-structures.

7.1 U(1) and the Central Trivector

The group $U(1)$ represents the Abelian phase rotation symmetry (Hypercharge). In the geometric algebra of the null substrate $V_3^{\mathbf{c}}$, the generator for this symmetry must commute with all other elements of the algebra (the $SU(2)$ and $SU(3)$ generators).

This identifies the $U(1)$ generator uniquely as the **Central Trivector** (Volume Element) of the simplex:

$$\Omega = \mathbf{c}_1 \wedge \mathbf{c}_2 \wedge \mathbf{c}_3 \tag{14}$$

Since the dimension of the simplex is odd ($n = 3$), the trivector lies in the center of the algebra. We define the geometric imaginary unit I by normalizing the volume element such that $I^2 = -1$:

$$I = \frac{\Omega}{|\Omega|} \quad \implies \quad I^2 = -1 \tag{15}$$

Thus, the electromagnetic phase $e^{I\theta}$ is a rotation in the total volume of the color space, commuting with the internal flavor dynamics, exactly matching the physical properties of the $U(1)$ gauge field.

7.2 SU(2) and Weak Isospin

The group $SU(2)$ describes the weak interaction and is isomorphic to the group of unit quaternions. In the QSNV framework, this is an immediate consequence of the **Recursive Orthogonalization Theorem** [1] applied to the u, d sub-simplex.

Identifying the quarks with $\mathbf{c}_1(u)$ and $\mathbf{c}_2(d)$, we observe that the physical mesons align with the orthogonal basis vectors $\mathbf{f}_k$ derived in our companion paper:

1. **The Pion (π^+):** Corresponds to the first recursive vector $\mathbf{f}_1 = \mathbf{c}_1 - \mathbf{c}_2$. As established, $\mathbf{f}_1^2 = -1$.

2. **The Isospin Altitude:** The vector orthogonal to the pion is the second recursive vector $\mathbf{f}_2 = \mathbf{c}_3 - (\mathbf{c}_1 + \mathbf{c}_2)$.

Since $\{\mathbf{f}_1, \mathbf{f}_2\}$ are mutually orthogonal and both square to -1, they generate the geometric algebra $\mathbb{G}_{0,2} \cong \mathbb{H}$ (Quaternions). Thus, the Lie algebra of the $u - d - s$ simplex naturally contains the $\mathfrak{su}(2)$ subalgebra generated by the bivector $\mathbf{f}_1\mathbf{f}_2$, driving the flavor dynamics without requiring ad-hoc Pauli matrices.

7.3 SU(3) and Strong Interactions

The group $SU(3)$ is the symmetry group of the strong interaction (QCD). As shown in Section 4, this is the natural symmetry group of the full Null Simplex $V_3^{\mathbf{c}}$. The 8 generators of $SU(3)$ (the Gell-Mann matrices) map directly to the 8 geometric elements of the substrate:

- 2 Cartan Generators (Diagonal): Correspond to the interaction bivectors $\mathbf{c}_1 \wedge \mathbf{c}_2$ and $\mathbf{c}_2 \wedge \mathbf{c}_3$.

- 6 Root Vectors (Off-Diagonal): Correspond to the meson difference vectors $\mathbf{c}_i - \mathbf{c}_j$.

Thus, the "Eightfold Way" is simply the complete set of geometric connections within a single tripartite null simplex.

8 Physical Interpretation: The Geometry of Symmetry

The QSNV framework resolves a long-standing tension between spacetime geometry (typically hyperbolic) and internal gauge symmetries (typically compact).

8.1 Corollary: The Separation of Compact and Hyperbolic Sectors

Corollary 1 (Generator Signature). The Lie Algebra of the Null Simplex splits naturally into compact and non-compact sectors based on the choice of basis interaction.

1. **The Hyperbolic Sector (Spacetime/Mixing):** The interaction bivectors of the null basis $\mathbf{L}_{ij} = \mathbf{c}_i \wedge \mathbf{c}_j$ satisfy:

$$\mathbf{L}_{ij}^2 = (\mathbf{c}_i \wedge \mathbf{c}_j)^2 = \frac{1}{4} > 0 \tag{16}$$

Since the square is positive, $e^{\mathbf{L}_{ij}\phi}$ generates non-compact hyperbolic rotations (boosts) [4]. These correspond to the mixing sectors of the Standard Model (e.g., the Weinberg angle) or the connection to the embedding spacetime.

2. **The Compact Sector (Flavor/Color):** The interaction bivectors of the recursive root basis $\mathbf{I}_k = \mathbf{f}_k \wedge \mathbf{f}_{k+1}$ (derived in Section 7.2) satisfy:

$$\mathbf{I}_k^2 = -1 \tag{17}$$

Since the square is negative, $e^{\mathbf{I}_k\theta}$ generates compact unitary groups ($SU(2), SU(3)$). This provides the rigorous geometric foundation for conserved quantum numbers like Isospin and Color.

8.2 Summary of Generators and Permutations

The generators of the unitary groups act on the simplex volume $\Omega = \mathbf{c}_1 \wedge \mathbf{c}_2 \wedge \mathbf{c}_3$ to produce the characteristic permutations of the particle multiplets.

Table 1: Geometric Origins of Standard Model Symmetries

Group	**Geometric Generator**	S_3 **Permutation**	**Physical Effect**
$U(1)$	Volume $\Omega = \mathbf{c}_1 \wedge \mathbf{c}_2 \wedge \mathbf{c}_3$	Identity ()	Global Phase (Hypercharge)
$SU(2)$	Root Plane $\mathbf{f}_1 \wedge \mathbf{f}_2$	Transposition (1 2)	$u \leftrightarrow d$ Swap (Isospin)
$SU(3)$	Full Simplex $\sum \mathbf{f}_i \wedge \mathbf{f}_j$	Full S_3 Group	Flavor/Color Mixing

9 Conclusion

The $SU(3)$ classification of hadrons is not an arbitrary imposition of complex group theory, but the direct geometric consequence of interactions within a real Null Simplex. By analyzing the substrate through the lens of the **Recursive Orthogonalization Theorem**, we find that:

1. The **Generations of Matter** are the manifestations of the Triality automorphism inherent in the Fano Plane structure of the substrate.

2. The **Gauge Groups** naturally separate into hyperbolic (spacetime mixing) and compact (particle flavor) sectors based on the choice of null versus root basis.

Finally, while this paper focuses on the specific application to the Standard Model, the rigorous mathematical foundation for treating complex Hermitian spaces $\mathbb{H}^{p,q}$ entirely within real geometric algebra is comprehensively established in the author's textbook, *New Foundations in Mathematics: The Geometric Concept of Number* [2]. Specifically, Chapter 10 of that work details the construction of the Unitary Geometric Algebra $\mathcal{U}_{p,q}$, providing the necessary theorems to rigorously justify the complex-real unification employed here. Readers are encouraged to consult this text for the complete mathematical theory underlying these physical results.

References

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